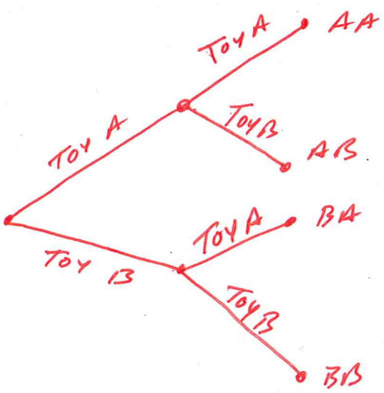
10.2 ~ Counting Outcomes and Tree Diagrams

Daily Objectives:

1. Use tree diagrams as an aid to counting possibilities for compound events.
2. Use the multiplication rule for independent events.
3. Explore conditional probability.

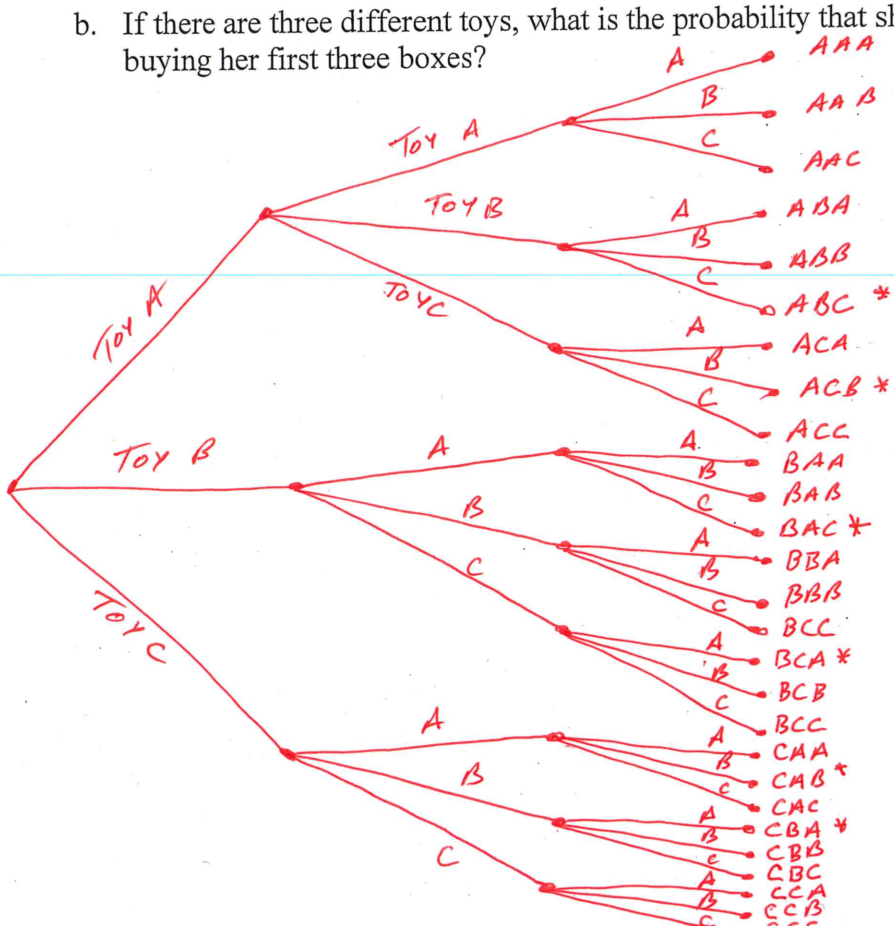
Example 1: A national advertisement says that every puffed-barley cereal box contains a toy and that the toys are distributed equally. Talya wants to collect a complete set of the different toys from cereal boxes.

a. If there are two different toys, what is the probability that she will find both of them in her first two boxes?



$\frac{2}{4} = \frac{1}{2} = .5$

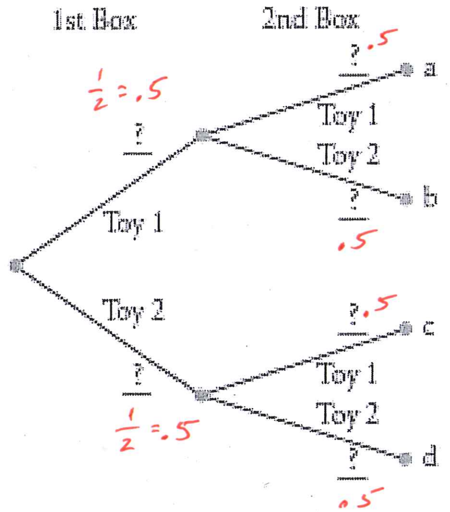
b. If there are three different toys, what is the probability that she will have them all after buying her first three boxes?



$\frac{6}{27} = \frac{2}{9} = .\overline{2}$

The Multiplication Rule Investigation:

Step 1: Below is the tree diagram from Example 1 part (a). This time write the probability on each branch. Then find the probability of each path.



P(Toy 1 followed by Toy 1) = ? $\frac{1}{4} = .25$

P(Toy 1 followed by Toy 2) = ? $\frac{1}{4} = .25$

P(Toy 2 followed by Toy 1) = ? $\frac{1}{4} = .25$

P(Toy 2 followed by Toy 2) = ? $\frac{1}{4} = .25$

What is the sum of the probabilities of all possible paths? 1.0

What is the sum of the probabilities of all the highlighted paths? .25 + .25 = .5

Step 2: Suppose the national advertisement mentioned in Example 1 listed four different toys distributed equally in a huge supply of boxes. Draw only as much of a tree diagram as you need in order to answer the following questions:

a. What would P(toy 2) in Tayla's first box? Talya's second box? Third box?

$\frac{1}{4} = .25$ $\frac{1}{4} = .25$ $\frac{1}{4} = .25$

b. In these situations, does the toy she finds in one box influence the probability of their being a particular toy in the next box?

NO

c. One outcome that includes all four toys is Toy 3, followed by Toy 2, followed by Toy 4, followed by Toy 1. What is the probability of this outcome?

$\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{256}$

$(.25)^4 = \boxed{.0039}$

d. Another outcome would be the same four toys in a different order. How many such outcomes are there? # OF WAYS 1st CAN OCCUR × # of WAYS 2nd × 3rd × 4th

$4 \times 4 \times 4 \times 4 = \boxed{256}$

Step 3: Write a statement explaining how to use the probabilities of a path's branches to find the probability of the path. The probabilities of each branch in a path can be multiplied to find the probabity of a particular path.

Step 4: What is the P(obtaining the complete set in the first four boxes)?

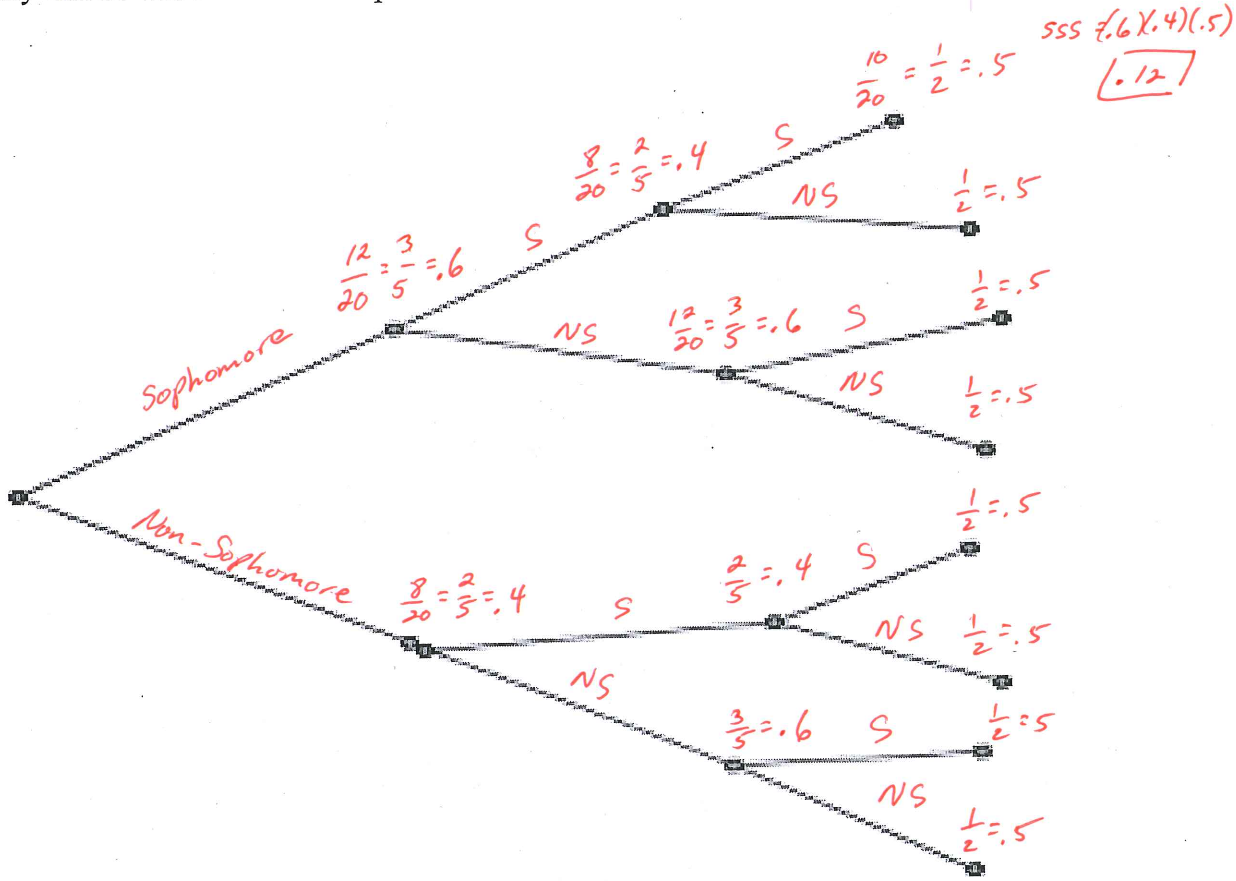
ABCD
ABDC
ACBD
ACDB
ADBC
ADCB

6 choosing A first

$6 \times 4 = 24$

$\frac{24}{256} = .09375$

Example 2: Mr. Roark teaches three classes. Each class has 20 students. His first class has 12 sophomores, his second class has 8 sophomore, and his third class has10 sophomores. If he randomly chooses one student from each class to participate in a competition what is the probability that he will select three sophomores?



Independent: Events are independent when the occurrence of one has NO influence on the probability of another.

Probability of a Path
(The Multiplication Rule for Independent Events)

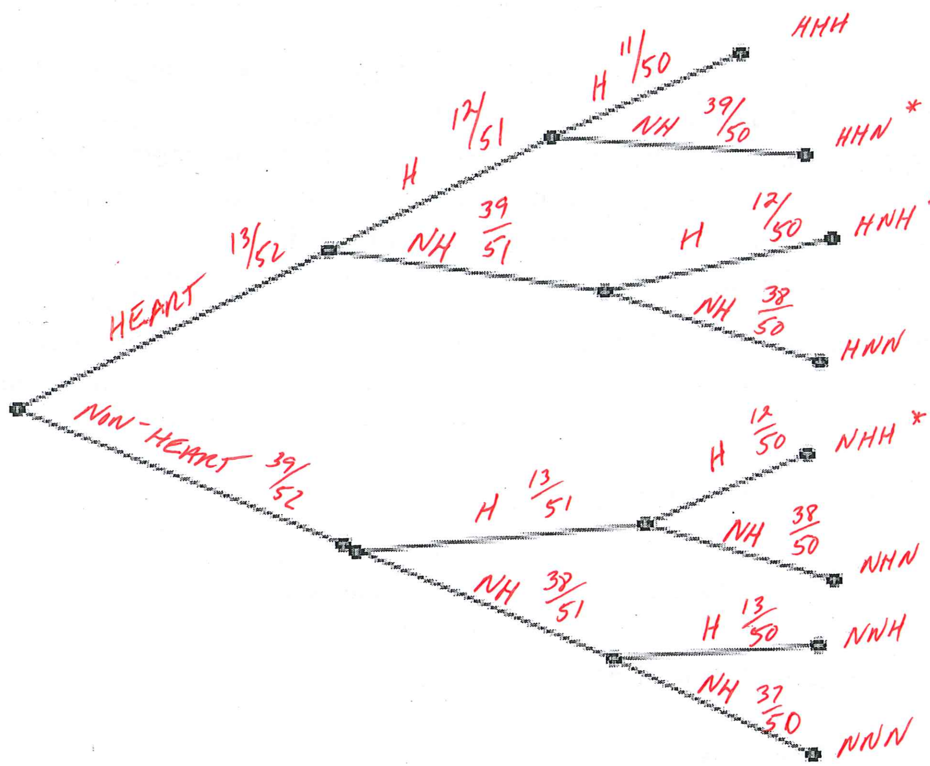
If n_1, n_2, n_3, and so on, represent independent events, then the probability that this sequence of events will occur can be found by multiplying the probabilities of the events.

$$P(n_1 \text{ and } n_2 \text{ and } n_3 \text{ and} \ldots) = P(n_1) \cdot P(n_2) \cdot P(n_3) \cdot \cdots$$

Example 3: A bag contains 3 orange blocks, 5 green blocks, and 7 yellow blocks. What is the probability that you will pick two green blocks in a row? [note: after your first draw, you put that respective block back in the bag].

$$P(GG) = P(G) \cdot P(G) = \frac{5}{15} \cdot \frac{5}{15} = \frac{1}{3} \cdot \frac{1}{3} = \boxed{\frac{1}{9} = .\overline{1}}$$

Example 4: Devon is going to draw three cards, one after another, from a standard deck. What is the probability that she will draw exactly two hearts?



HHN $* \; \frac{1}{4} \cdot \frac{4}{17} \cdot \frac{39}{50} = .04588235$

HNH $* \; \frac{1}{4} \cdot \frac{13}{17} \cdot \frac{6}{25} = .04588235$

NHH $* \; \frac{39}{52} \cdot \frac{13}{51} \cdot \frac{6}{25} = .04588235$

$\boxed{\approx .1376}$

Dependent Events: Events are dependent when the probability of occurrence of one event *depends* on the occurrence of another.

The Multiplication Rule (again)

If n_1, n_2, n_3, and so on, represent events, then the probability that this sequence of events will occur can be found by multiplying the conditional probabilities of the events.

$$P(n_1 \text{ and } n_2 \text{ and } n_3 \text{ and} \ldots) = P(n_1) \cdot P(n_2 \mid n_1) \cdot P(n_3 \mid (n_1 \text{ and } n_2)) \cdot \cdots$$

Conditional Probability: The probability of a particular dependent event, given the outcome of an event on which it depends.

If events A and B are independent, then the probability of A is the same whether B happens or not. In this case, $P(A \mid B) = P(A)$. In Example B, the probability of choosing a sophomore in the second class (S2) is the same whether Mr. Roark chooses a sophomore or a nonsophomore from his first class. This means that $P(S2 \mid S1) = P(S2 \mid NS1) = P(S2)$. Can you explain the difference in the multiplication rule defined on page 563 and the rule defined above?

This tree could represent any two-stage event with two options at each stage. To find the probability of event C, you must add all paths, or outcomes, that contain C. $P(C) = P(A \text{ and } C) + P(B \text{ and } C) = P(A) \cdot P(C \mid A) + P(B) \cdot P(C \mid B)$.